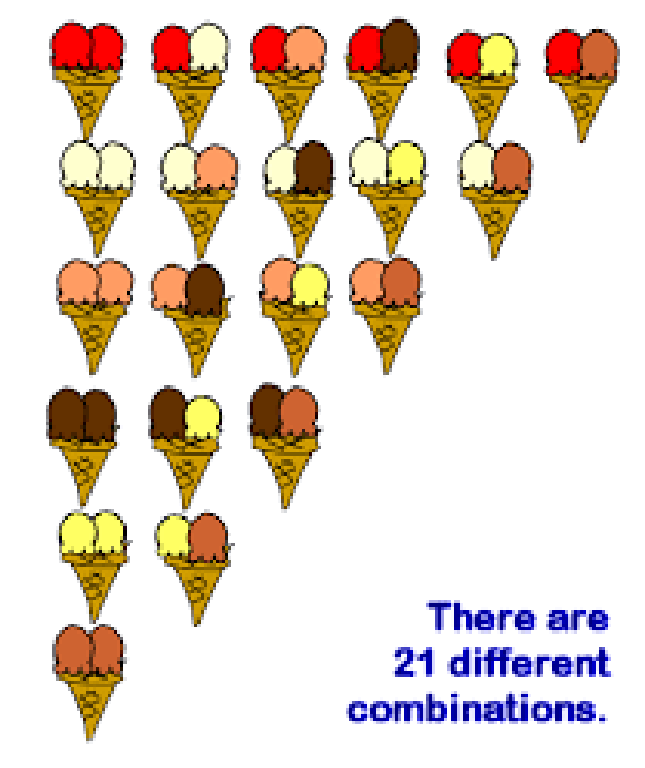
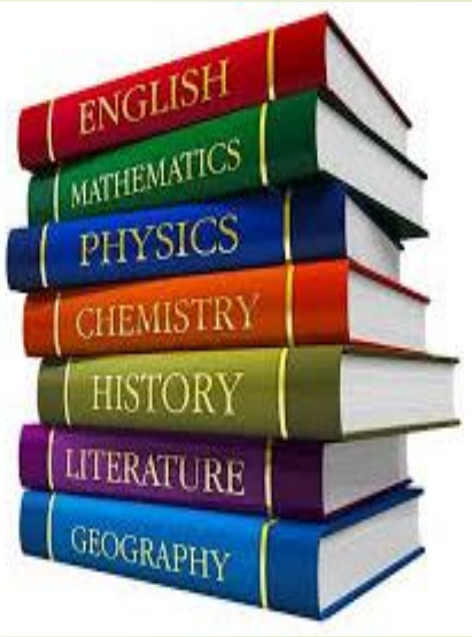
COMBINATIONS.....

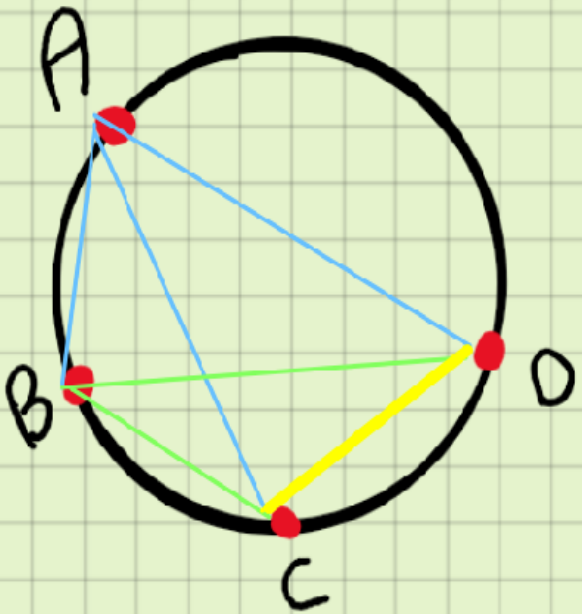
MODULE-8

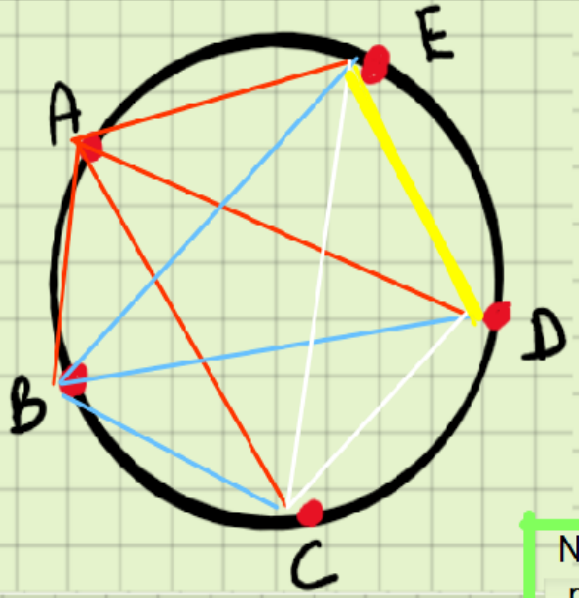


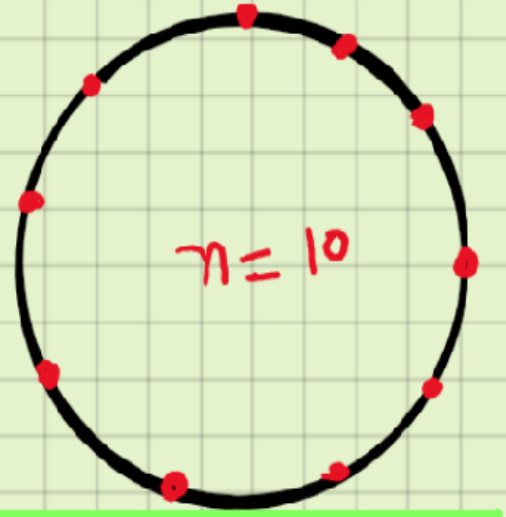


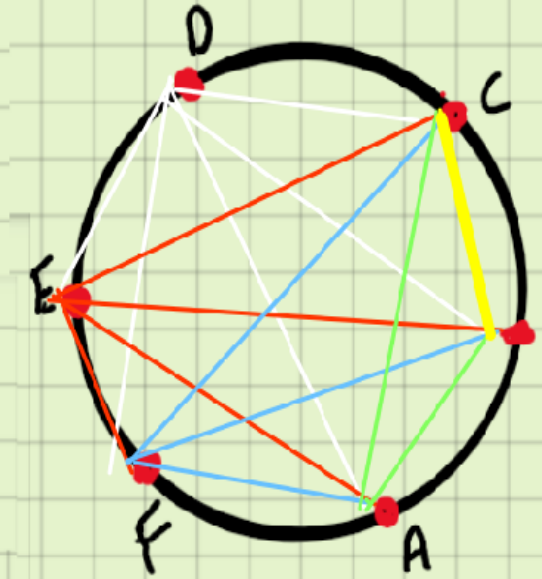
Number of lines that can be drawn through some points on the circle

1. WHY CIRCLE ?
2. WHY NOT PLANE ?









No. of points	Total lines	Deduction
4	6	3, 2, 1
5	10	4, 3, 2, 1
6	15	5, 4, 3, 2, 1
10	45	9, 8, 7, 6, 5, 4, 3, 2, 1

Can you deduce the result for n points

No. of lines that can be drawn .....

- When n=4, no. of lines drawn =3+2+1 = 6
- When n=5, no. of lines drawn

  = 4+3+2+1= 10
- When n=6, no. of lines drawn

  = 5+4+3+2+1 = 15
- When n=10. no. of lines drawn

  = 9+8+7+6+5+4+3+2+1= 45

So, In general, when there are 'n' points on a circle, no. of lines drawn

=(n-1)+(n-2)+(n-3)+…..+3+2+1

= sum of first (n-1) natural numbers

$= \frac{n(n-1)}{2} = {}^{n}C_{2}$

Q) How many chords can be drawn through 21 points on a circle?

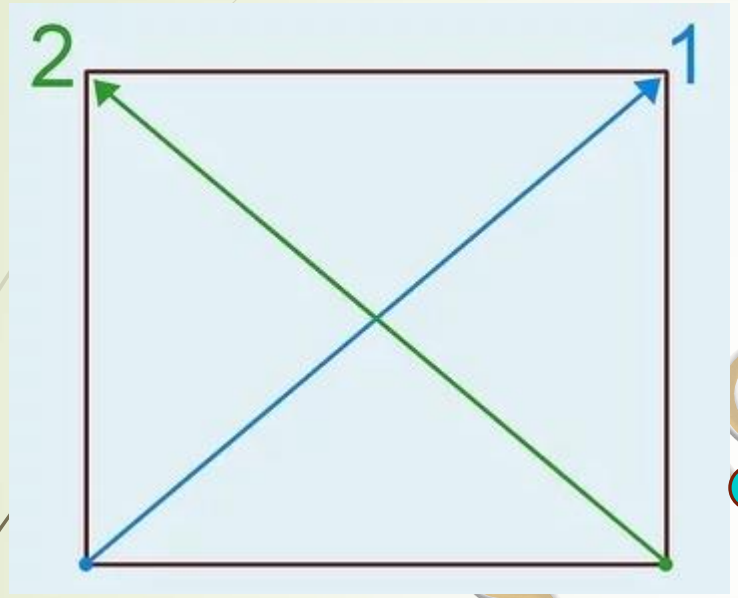
Ans) There are 21 points on the circle. Since only one chord can be drawn by joining 2 distinct points, so the required no. of chords is:

$${}^{21}C_{2} = \frac{21!}{19!2!} = \frac{21 \times 20}{2} = 210$$

If there are 38 points ????

NUMBER OF DIAGONALS OF A POLYGON HAVING:

- **4 SIDES** : (4 vertices)



For a line segment, 2 points are needed

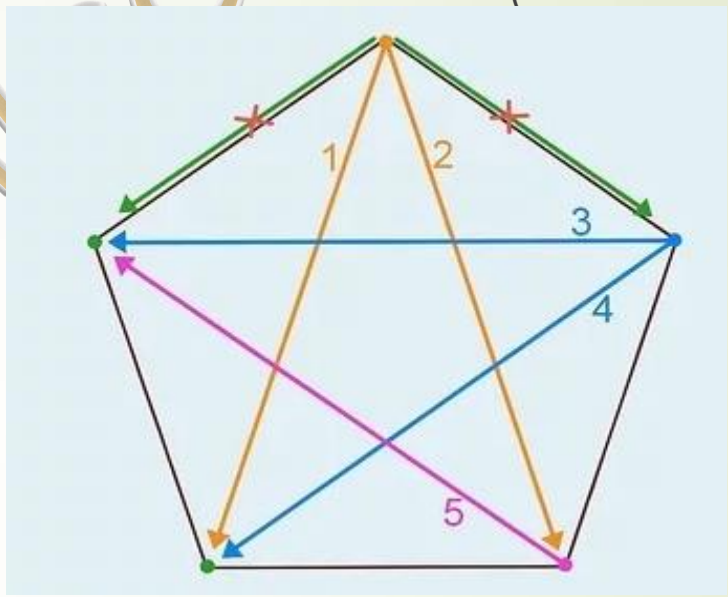
- Total number of line segments formed = $^4C_2 = 6$

But

Number of sides = 4. Here, n = 4

$\Rightarrow$ Number of diagonals = 6-4 =2

- **5 SIDES** : (5 vertices)



- Total number of line segments formed = $^5C_2 = 10$

But

Number of sides = 5. Here, n = 5

$\Rightarrow$ Number of diagonals = 10 – 5 = 5

Number of diagonals in a polygon…..

- Now…. If n = 8, then,

No. of diagonals = $^{8}C_{2} - 8$

$= \frac{8 \times 7}{2} - 8 = 20$

For n = 20, number of diagonals will be _____ ????

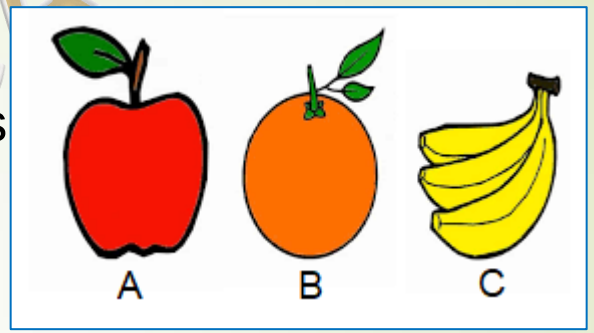
- So, in general, if a polygon has ‘n’ sides, then the number of diagonals is given by,

$^{n}C_{2} - n$

Let's consider an Example……

There are 4 marker pens

and 3 fruits



selection of a marker → 4 ways
selection of a fruit → 3 ways

? In how many different ways can one marker AND one fruit be selected? (From both)

→ No. of selections = 4 **x** 3 =12 $\left[\begin{matrix} P_1A,\ P_2A,\ P_3A,\ P_4A,\ P_1B,\ P_2B, P_3B, \\ P_4B, P_1C,\ P_2C,\ P_3C,\ P_4C \end{matrix}\right]$ using FPC

? In how many different ways can one marker OR one fruit be selected? (From any)

→ No. of selections = 4 **+** 3 = 7 $[P_1, P_2, P_3, P_4, A, B, C]$

Cards (52)

Face cards are King + Queen + Jack

So, Total Face cards = 3 × 4 = 12 cards

Spade ♠	Club ♣	Diamond ♦	Heart ❤
1 King	1 King	1 King	1 King
1 Queen	1 Queen	1 Queen	1 Queen
1 Jack	1 Jack	1 Jack	1 Jack
1 Ace	1 Ace	1 Ace	1 Ace
2-10 Cards	2-10 Cards	2-10 Cards	2-10 Cards
Total = 13	**Total** = 13	**Total** = 13	**Total** = 13

Find the number of ways of choosing 4 cards from a deck of 52 playing cards? In how many of these :

No. of ways of choosing 4 cards from a deck = $^{52}C_4$

(i) four cards are of the same suit:

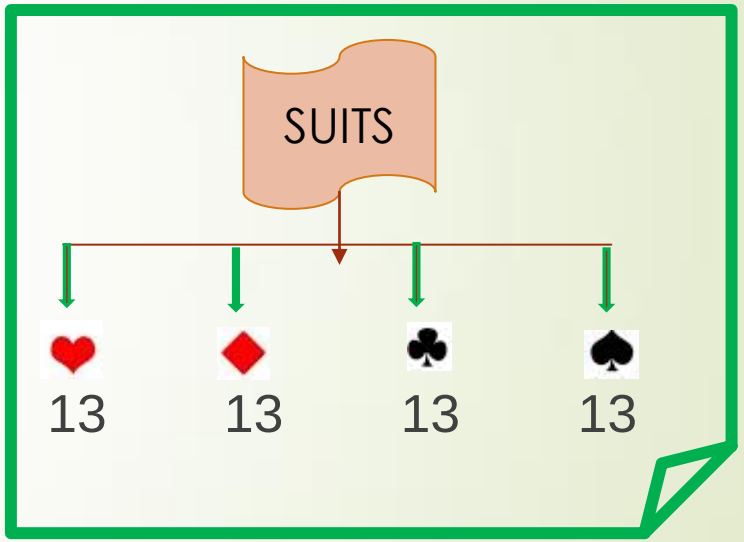
$= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$

$= 4x \frac{13!}{4!\ 9!} = 2860.$

OR → +
AND → x

(ii) four cards belong to four different suits:

$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = 13^4$



(iii) two are red cards and two are black cards:

$= {}^{26}C_2 \times {}^{26}C_2 = 105625$

26 → RED
26 → BLACK

(iv) cards are of the same colour:

$= {}^{26}C_4 + {}^{26}C_4 = 2 \times \frac{26!}{4!\ 22!} = 29900.$

(v) all four are face cards:

$= {}^{12}C_4 = \frac{12!}{4!\ 8!} = 495.$

Face cards are King + Queen + Jack

So, Total Face cards = 3 × 4 = 12 cards

(vi) there is exactly one ace :

$= {}^{4}C_1 \times {}^{48}C_3 = 4 \times \frac{48!}{45!\ 3!} = 69184.$

Ace Cards → 4
Remaining → 48

POLL QUESTION

POLL QUESTION.....

- A polygon has 27 diagonals. Find the number of its sides.

(A) 6 (B) 7 (C) 8 (D) 9

HOMEWORK QUESTIONS…..

1) A polygon has 44 diagonals. Find the number of its sides.

2) How many triangles can be formed by joining the vertices of an octagon?

3) How many chords can be drawn through 28 points on a circle?

4) Twelve persons meet in a room and each shakes hand with all others. Find the number of handshakes.

5) Determine ‘n’ if $^{2n}C_3 : {}^{n}C_3 = 11 : 1$

6) Find ‘n’ if $^{24}C_n = {}^{24}C_{2n+3}$

ANSWERS:

1) 11 2) 56 3) 378 4) 66 5) 6 6) 7